Corrections to Theory of Asset Pricing (2008), Pearson, Boston, MA

1. Page 7. Revise the Independence Axiom to read:

   For any two lotteries P and P^*, $P^* \succ P$ if and only if for all $\lambda \in$ (0,1] and all P^{**}:

   $$\lambda P^* + (1-\lambda)P^{**} \succ \lambda P + (1-\lambda)P^{**}$$

   Moreover, for any two lotteries P and $P^\dagger$, $P \sim P^\dagger$ if and only if for all λ $\in$(0,1] and all P^{**}:

   $$\lambda P + (1-\lambda)P^{**} \sim \lambda P^\dagger + (1-\lambda)P^{**}$$

2. Page 29. Revise the last paragraph to read:

   Next, suppose that utility is not quadratic but any general increasing, concave form. Are there particular probability distributions for portfolio returns that make expected utility, again, depend only on the portfolio return's mean and variance? Such distributions would need to be fully determined by their means and variances, that is, they must be two-parameter distributions whereby higher-order moments could be expressed in terms of the first two moments (mean and variance). Many distributions, such as the gamma, normal, and lognormal, satisfy this criterion. But in the context of an investor's portfolio selection problem, such distributions need to satisfy other reasonable conditions.

   Since an individual is able to choose which assets to combine into a portfolio, all portfolios created from a combination of individual assets or other portfolios must have distributions that continue to be determined by their means and variances. In other words, we need a distribution such that if the individual assets' return distributions depend on just mean and variance, then the return on a linear combination (portfolio) of these assets has a distribution that depends on just the portfolio's mean and variance. Furthermore, the distribution should allow for a portfolio that possibly includes a risk-free (zero variance) asset, as well as assets that may be independently distributed. The only distributions that satisfy these "additivity," "possible risk-free asset," and "possible independent assets" restrictions is the stable family of distributions.[1] However, the only distribution within the stable family that has finite variance is the normal (Gaussian) distribution. Thus, since the multivariate normal distribution satisfies these portfolio conditions and has finite variance, it can be used to justify mean-variance analysis.

3. Page 33. Revise equation (2.9) to add subscripts.

$$U'(\bar{R}_p + x_i\sigma_p) < U'(\bar{R}_p - x_i\sigma_p) \tag{2.9}$$

4. Page 53 Below equation (2.66), the definition of Δp_i^s should be $\Delta p_i^s \equiv p_{i1}^s - p_{i0}^s$, $i = 1,..., n$.

5. Page 63 equation (3.11). Change N to n:

$$\frac{\partial \sigma_m}{\partial \omega_i^m} = \frac{1}{2\sigma_m}\frac{\partial \sigma_m^2}{\partial \omega_i^m} = \frac{1}{2\sigma_m}\frac{\partial \omega^m V \omega^m}{\partial \omega_i^m} = \frac{1}{2\sigma_m}2V_i\omega^m = \frac{1}{\sigma_m}\sum_{j=1}^{n}\omega_j^m \sigma_{ij} \tag{3.11}$$

6. Page 73. To indicate that it is a column vector, add a prime after $b_z = [b_{1z}\ b_{2z}\ ...\ b_{nz}]'$, $z = 1, ..., k$.

7. Page 93. It is clearer to write equation (4.41) as

$$e_s = \begin{bmatrix} W_1 \\ \vdots \\ W_{s-1} \\ W_s \\ W_{s+1} \\ \vdots \\ W_k \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{4.41}$$

8. Page 137. In Exercise 4 part a., it should read "Show whether or not $p_t = f_t + b_t$, subject to the specifications in (6.39) and (6.40), is a valid solution for the price of the risky asset."

9. Page 165. In footnote 1, revise the sentence "We examine how to model an asset price process that is a mixture of a diffusion process and a jump process in Chapter 11."

[1] See Chamberlain (1983) and Liu (2004).

10. Page 167 to 168. It should be noted that there may be other probability limits, in addition to the normally-distributed Brownian motion, for processes that have independent and identically distributed increments. For example, a compensated Poisson process can have zero mean and variance proportional to time but would not have a limiting distribution that is Brownian motion. To obtain normality, we also need to assume a continuous sample path. See Merton (1990, Chapter 3).

11. Page 185. Equation (9.13) has an extraneous μ. It should read

$$dH(t) = \left[\sum_{i=1}^{n} \omega_i(t)(\mu_i - r)H(t) \;+ rH(t) - F(t)\right] dt + \sum_{i=1}^{n} \omega_i(t) H(t) \sigma_i \, dz_i \tag{9.13}$$

12. Page 193. Equation (9.40) has an extraneous α. It should read

$$\tfrac{\sigma_r^2}{2} P_{rr} + \left[\alpha(\bar{r} - r) + q\sigma_r\right] P_r - rP - P_\tau = 0 \tag{9.40}$$

13. Page 200. Exercise 2 should state "where α, $\bar{r}$, and σ are positive constants."

14. Page 207. In equation (10.12), s should be changed to u :

$$\xi_\tau = \exp\left[-\int_0^\tau \theta(u)\, dz - \frac{1}{2}\int_0^\tau \theta(u)^2 \, du\right] \tag{10.12}$$

15. Page 233. Under equation (11.23), the definition should be $r_n \equiv r - \lambda k + n\alpha/\tau$.

16. Page 255. Before equation (12.42), it should be $C_t = \frac{\gamma}{1-\gamma}\left[\frac{\rho}{\gamma} - r - \frac{(\mu-r)^2}{2(1-\gamma)\sigma^2}\right] W_t$ $= \frac{\gamma}{1-\gamma}\left[\frac{\rho}{\gamma} - r - \frac{\left(\sum_{i=1}^{n} \delta_i \mathrm{p}_i\right)^2}{2(1-\gamma)\sigma^2}\right] W_t,$

17. Page 257. The last term after the first equality in equation (12.45) should have a minus sign and, therefore, the last term after the second equality in

equation (12.45) will have a plus sign:

$$
\begin{aligned}
0 &= U\left(C_t^*, t\right) + J_t + J_W\left[rW_t - C_t^*\right] + a\left(x,t\right) J_x \\
&\quad + \frac{1}{2} b\left(x,t\right)^2 J_{xx} - \frac{J_W^2}{2J_{WW}} \sum_{i=1}^{n}\sum_{j=1}^{n} \upsilon_{ij}\left(\mu_j - r\right)\left(\mu_i - r\right) \\
&= e^{-\rho t}\ln\left[\frac{W_t}{d\left(t\right)}\right] + e^{-\rho t}\left[\frac{\partial d\left(t\right)}{\partial t} - \rho d\left(t\right)\right]\ln\left[W_t\right] + F_t + e^{-\rho t} d\left(t\right) r - e^{-\rho t} \\
&\quad + a\left(x,t\right) F_x + \frac{1}{2} b\left(x,t\right)^2 F_{xx} + \frac{d\left(t\right) e^{-\rho t}}{2} \sum_{i=1}^{n}\sum_{j=1}^{n} \upsilon_{ij}\left(\mu_j - r\right)\left(\mu_i - r\right)
\end{aligned}
\tag{12.45}
$$

Similarly, this makes the last terms in equations (12.46) and (12.49) to have plus signs:

$$
\begin{aligned}
0 &= -\ln\left[d\left(t\right)\right] + \left[1 + \frac{\partial d\left(t\right)}{\partial t} - \rho d\left(t\right)\right]\ln\left[W_t\right] + e^{\rho t} F_t + d\left(t\right) r - 1 \\
&\quad + a\left(x,t\right) e^{\rho t} F_x + \frac{1}{2} b\left(x,t\right)^2 e^{\rho t} F_{xx} + \frac{d\left(t\right)}{2} \sum_{i=1}^{n}\sum_{j=1}^{n} \upsilon_{ij}\left(\mu_j - r\right)\left(\mu_i - r\right)
\end{aligned}
\tag{12.46}
$$

$$
\begin{aligned}
0 &= -\ln\left[d\left(t\right)\right] + e^{\rho t} F_t + d\left(t\right) r - 1 + a\left(x,t\right) e^{\rho t} F_x \\
&\quad + \frac{1}{2} b\left(x,t\right)^2 e^{\rho t} F_{xx} + \frac{d\left(t\right)}{2} \sum_{i=1}^{n}\sum_{j=1}^{n} \upsilon_{ij}\left(\mu_j - r\right)\left(\mu_i - r\right)
\end{aligned}
\tag{12.49}
$$

18. Page 286 The first line of equation (13.33) should be

$$
\Psi_i \equiv \frac{\partial J}{\partial W}\mu_i W + \frac{\partial^2 J}{\partial W^2}\sum_{j=1}^{n}\sigma_{ij}\omega_j^* W^2 + \sum_{j=1}^{k}\frac{\partial^2 J}{\partial x_j \partial W}\phi_{ij} W - \lambda \leq 0
$$

19. Page 301. The exponent of the first term in (14.21) should have a minus sign, so that (14.21) is corrected to read

$$
\frac{1-\gamma}{\gamma}\left[J_W - J_x\right]^{\frac{-\gamma}{1-\gamma}} - \frac{J_W^2}{J_{WW}}\frac{(\mu - r)^2}{2\sigma^2} + (rW - bx)J_W + (b-a)xJ_x - \rho J = 0
$$

20. Page 310. In the second paragraph, the reference to (14.58) should be (14.57): "...the utility function given in (14.56) and (14.57) is (ordinally) equivalent..."

21. Page 312. In the first sentence, change "partial differential equation" to "ordinary differential equation."

22. Page 313 The first three sentences on this page should read "Similar to the time-separable case, for an infinite horizon solution to exist, we need consumption to be positive in (14.70), which requires

$$\rho > \frac{\epsilon - 1}{\epsilon} \left(r + [\mu - r]^2 / \left[2 (1 - \gamma) \sigma^2 \right] \right).$$

This will be the case when the elasticity of intertemporal substitution, ϵ, is sufficiently small. For example, assuming $\rho > 0$, this inequality is always satisfied when $\epsilon < 1$.

23. Page 315. In equation (14.76), $+\gamma/\epsilon$ should be $-\gamma/\epsilon$:

$$r = \rho + \frac{g_c}{\epsilon} - \left[1 - \gamma - \frac{\gamma}{\epsilon} \right] \frac{\sigma_c^2}{2}$$

Also change the last sentence in Section 14.3 from "From (14.76) we see that a value of $\epsilon = 1$ would make r independent of risk aversion, γ, and, assuming ρ is small, could produce a reasonable value for the real interest rate." to "From (14.76) we see that since $\sigma_c^2/2$ is small and assuming ρ is also small, a value of $\epsilon = 1$ could produce a reasonable value for the real interest rate.

24. Page 324. Correct the definition of w_t to be "w_t denotes the number of shares of the risky asset held by the individual at date t."

25. Page 325. In the first paragraph below equation (15.5), the fourth sentence should be corrected to read "Conversely, when $\eta = 1$ but $R_t < \overline{R}$, z_t is larger than z_{t-1}."

26. Page 335. The last term in equation (15.47) should be corrected to be $= S_{r,t} e^{\eta \sigma^2 (T-t)}$.

27. Page 338. Insert “expected” in the sentence “This portfolio policy is referred to as the “growth-optimum portfolio,” because it maximizes the expected (continuously compounded) return on wealth.”

28. Page 348. In the fourth paragraph, correct the variance so that the sentence reads “The equilibrium would be the same if all traders received the same signal, $\bar{y} \sim N(m, \sigma^2 + \frac{\sigma_\epsilon^2}{n})$ or if they all decided to share information on their private signals among each other before trading commenced.”

29. Page 350. In equations (16.19), (16.21), and in the text below (16.21), remove the extraneous right parenthesis to change “$\text{Var}\left[\tilde{P}_1 \mid I_i\,)\right]$” to “$\text{Var}\left[\tilde{P}_1 \mid I_i\right]$.”

References

[1] Chamberlain, G. (1983): “A Characterization of the Distributions That Imply Mean-Variance Utility Functions,” *Journal of Economic Theory*, 29, 185-201.

[2] Liu, L. (2004): “A New Foundation for the Mean-Variance Analysis,” European Journal of Operational Research, 158, 229-242.

[3] Merton, R.C. (1990): *Continuous-Time Finance*. Blackwell, Cambridge, MA.