

Theory of Asset Pricing

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Preface

The genesis of this book comes from my experience teaching asset pricing theory to beginning doctoral students in finance and economics. What I found was that no existing text included all of the major theories and techniques of asset valuation that students studying for a Ph.D. in financial economics should know. While there are many excellent books in this area, none seemed ideal as a stand-alone text for a one-semester first course in theoretical asset pricing. My choice of this book's topics were those that I believe are most valuable to someone at the start of a career in financial research. Probably the two features that most distinguish this book from others are its broad coverage and its user-friendliness.

Contents of this book have been used for over a decade in introductory finance theory courses presented to doctoral students and advanced masters students at the University of Illinois at Urbana-Champaign. The book presumes students have a background in mathematical probability and statistics and that they are familiar with constrained maximization (Lagrange multiplier) problems. A prior course in microeconomics at the graduate or advanced undergraduate level would be helpful preparation for a course based on this book. However, I have found that doctoral students from mathematics, engineering, and the physical sciences who had little prior knowledge of economics often are able to understand the course material.

This book covers theories of asset pricing that are the foundation of current theoretical and empirical research in financial economics. It analyzes models of individual consumption and portfolio choice and their implications for equilibrium asset prices. In addition, contingent claims valuation techniques based on the absence of arbitrage are presented. Most of the consumption-portfolio choice models assume individuals have standard, time-separable expected utility functions, but the book also considers more recent models of utility that are not time separable or that incorporate behavioral biases. Further, while much of the analysis makes standard “perfect markets” assumptions, the book also examines the impact of asymmetric information on trading and asset prices. Many of the later chapters build on earlier ones, and important topics reoccur as models of increasing complexity are introduced to address them. Both discrete-time and continuous-time models are presented in a manner that attempts to be intuitive, easy to follow, and that avoids excessive formalism.

As its title makes clear, this book focuses on theory. While it sometimes contains brief remarks on whether a particular theory has been successful in explaining empirical findings, I expect that doctoral students will have additional exposure to an empirical investments seminar. Some of the material in the book may be skipped if time is limited to a one-semester course. For example, parts of Chapter 7’s coverage of binomial option pricing may be cut if students have seen this material in a masters-level derivatives course. Any or all of the chapters in Section V also may be omitted. In my teaching, I cover Chapter 15 on behavioral finance and asset pricing, in part because current research on this topic is expanding rapidly. However, if reviewer response is any indication, there are strongly held opinions about behavioral finance and asset pricing, and so I suspect some readers will choose to skip this material all together while others may wish to see it expanded.

Typically, I also cover Chapter 16 which outlines some of the important models of asymmetric information that I believe all doctoral students should know. However, many Ph.D. programs may offer a course entirely devoted to this topic, so that this material could be deleted under that circumstance. Chapters 17 and 18 on modeling default-free and defaultable bond prices contain advanced material that I typically do not have time to cover during a single semester. Still, there is a vast amount of research on default-free term structure models and a growing interest in modeling default risk. Thus, in response to reviewers' suggestions, I have included this material because some may find coverage of these topics helpful for their future research. A final note on the end of chapter problems: most of these problems derive from assignments and exams given to my students at the University of Illinois. The solutions are available for instructor download at the Addison Wesley website.

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